No cross-interactions between the Weyl graviton and the massless Rarita-Schwinger field

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Abstract

The proof of the fact that there are no nontrivial, consistent cross-couplings that can be added between the Weyl graviton and the massless Rarita-Schwinger field is accomplished by means of a cohomological approach, based on the deformation of the solution to the master equation from the antifield-Becchi-Rouet-Stora-Tyutin (BRST) formalism. The procedure developed here relies on the assumptions of locality, smoothness, (background) Lorentz invariance, Poincaré invariance, and preservation of the number of derivatives with respect to each field (the last hypothesis was made only in antighost number zero).

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1 Introduction

A key point in the development of local gauge field theory is represented, without any doubt, by the discovery of the Becchi-Rouet-Stora-Tyutin (BRST)

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symmetry [1, 2] and, in this context, by the cohomological reformulation of the antifield-BRST symmetry [3, 4, 5, 6, 7], which allowed the powerful algebraic methods to tackle many issues, like renormalization and anomalies [8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. The applications of the cohomological reformulation of the BRST approach cover a broad spectrum, including the reformulation of the problem of consistent interactions among gauge fields as a cohomological problem of deforming the solution to the master equation [18]. The main aim of this paper is to analyze the construction of consistent interactions that can be introduced between the linearized limit of Weyl gravity and the massless Rarita-Schwinger field from the BRST standpoint. Such an analysis is motivated on the one hand by the remarkable properties of conformal gravity and supergravity [19], as well as by the renewed interest in Weyl gravity [20] in connection with the ADS/CFT correspondence, and, on the other hand, by some supergravity models [21, 22, 23, 24], where the gravitino represents the supersymmetric partner of the graviton.

Our procedure is based on solving the equations that describe the deformation of the solution to the master equation by means of specific cohomological techniques. More precisely, we start from a free model describing the sum between the linearized limit of Weyl gravity and the massless Rarita-Schwinger action, and construct its antibracket-antifield BRST symmetry s, which splits as $s = \delta + \gamma$, where δ is the Koszul-Tate differential and γ represents the exterior longitudinal derivative. Next, we briefly review the basic equations of the antibracket-antifield deformation procedure, and then pass to solving the equation that describes the first-order deformation of the solution to the master equation. The local form of this equation shows that the nonintegrated density of the first-order deformation of the solution to the master equation belongs to the local BRST cohomology in ghost number zero $H^0(s|d)$. The core of the paper is then dedicated to the computation of $H^0(s|d)$. Based on this computation, we prove, under the assumptions of locality, smoothness, (background) Lorentz invariance, Poincaré invariance, and preservation of the number of derivatives with respect to each field (the last hypothesis was made only in antighost number zero), that there are no nontrivial, consistent cross-couplings that can be added between the Weyl graviton and the massless Rarita-Schwinger field. Consequently, the only interactions that can be added to the Lagrangian action are given by the self-interactions of the Weyl graviton, studied in detail in [25] from a cohomological perspective, since the massless Rarita-Schwinger field allows no consistent self-interactions, as it has been proved in [26] also on a coho-

mological basis. Our result is in agreement with the fact that it is not the Rarita-Schwinger field which appears in conformal supergravity, but a spin $\frac{3}{2}$ field, described in the free limit by the action [19]

$$S_0^{\frac{3}{2}}[\psi_\mu] \propto \int d^4x \left(\varepsilon^{\mu\nu\rho\lambda}\bar{\phi}_\rho\gamma_5\gamma_\lambda\partial_\mu\phi_\nu\right),$$

where

$$\phi_\mu = \tfrac{1}{3}\gamma^\sigma\left[\mathrm{i}\left(\partial_\sigma\psi_\mu - \partial_\mu\psi_\sigma\right) + \tfrac{1}{2}\gamma_5\varepsilon_{\sigma\mu\alpha\beta}\partial^\alpha\psi^\beta\right],$$

so it contains three derivatives instead of only one present in the Rarita-Schwinger theory.

2 Free model: Lagrangian formulation and BRST symmetry

Our starting point is represented by a free Lagrangian action written as the sum between the linearized Weyl gravity action [25] and the massless Rarita-Schwinger action [27]

$$S_0^L[h_{\mu\nu},\psi_\mu] = \tfrac{1}{2}\int d^4x \left(-\varepsilon^{\mu\nu\rho\lambda}\bar{\psi}_\mu\gamma_5\gamma_\nu\partial_\rho\psi_\lambda + \mathcal{W}_{\mu\nu\alpha\beta}\mathcal{W}^{\mu\nu\alpha\beta}\right), \tag{1}$$

where $\mathcal{W}_{\mu\nu\alpha\beta}$ is the linearized Weyl tensor in four spacetime dimensions, given in terms of the linearized Riemann tensor $\mathcal{R}_{\mu\nu\alpha\beta}$ and of its traces by

$$\mathcal{W}_{\mu\nu\alpha\beta} = \mathcal{R}_{\mu\nu\alpha\beta} - \tfrac{1}{2}\left(\sigma_{\mu[\alpha}\mathcal{R}_{\beta]\nu} - \sigma_{\nu[\alpha}\mathcal{R}_{\beta]\mu}\right) + \tfrac{1}{6}\mathcal{R}\sigma_{\mu[\alpha}\sigma_{\beta]\nu}. \tag{2}$$

Throughout the paper we work with the flat metric of 'mostly minus' signature $\sigma_{\mu\nu} = (+---)$. The notation $[\mu\ldots\nu]$ signifies full antisymmetry with respect to the indices between brackets without normalization factors (i.e. the independent terms appear only once and are not multiplied by overall numerical factors). The linearized Riemann tensor is expressed by

$$\begin{aligned}\mathcal{R}_{\mu\nu\alpha\beta} &= \tfrac{1}{2}\left(\partial_\mu\partial_\beta h_{\nu\alpha} + \partial_\nu\partial_\alpha h_{\mu\beta} - \partial_\nu\partial_\beta h_{\mu\alpha} - \partial_\mu\partial_\alpha h_{\nu\beta}\right)\\ &\equiv \tfrac{1}{2}\partial_{[\mu}h_{\nu][\alpha,\beta]},\end{aligned} \tag{3}$$

while its simple and respectively double traces read as

$$\mathcal{R}_{\mu\nu} = \sigma^{\alpha\beta}\mathcal{R}_{\mu\alpha\nu\beta}, \qquad \mathcal{R} = \sigma^{\mu\nu}\mathcal{R}_{\mu\nu}. \tag{4}$$

The linearized Weyl tensor can be expressed in terms of the symmetric tensor $\mathcal{K}_{\mu\nu}$ like

$$\mathcal{W}_{\mu\nu\alpha\beta} = \mathcal{R}_{\mu\nu\alpha\beta} - \left(\sigma_{\mu[\alpha}\mathcal{K}_{\beta]\nu} - \sigma_{\nu[\alpha}\mathcal{K}_{\beta]\mu}\right), \tag{5}$$

where

$$\mathcal{K}_{\mu\nu} = \tfrac{1}{2}\left(\mathcal{R}_{\mu\nu} - \tfrac{1}{6}\sigma_{\mu\nu}\mathcal{R}\right). \tag{6}$$

The spinor-vector ψ_μ has (Majorana) real components and the γ-matrices are in the Majorana representation

$$\gamma_\mu^* = -\gamma_\mu, \qquad \gamma_\mu^T = -\gamma_0\gamma_\mu\gamma_0, \qquad \left(\mu = \overline{0,3}\right), \tag{7}$$

where $*$ and T in (7) signifies the operations of complex conjugation and respectively of transposition. The theory described by (1) possesses an irreducible and abelian generating set of gauge transformations

$$\delta_{\epsilon,\theta}h_{\mu\nu} = \partial_{(\mu}\epsilon_{\nu)} + 2\sigma_{\mu\nu}\epsilon, \qquad \delta_{\epsilon,\theta}\psi_\mu = \partial_\mu\theta, \tag{8}$$

where the gauge parameters ϵ_μ and ϵ are bosonic and θ is a fermionic spinor with real components. The scalar gauge parameter ϵ is responsible for the so-called conformal invariance of Weyl theory. The notation $(\mu\nu)$ signifies symmetry with respect to the indices between parentheses without the factor $1/2$.

In order to construct the BRST symmetry for the model under study we introduce the ghosts η_μ, ξ and C respectively associated with the gauge parameters ϵ_μ, ϵ and θ. The ghosts η_μ and ξ are fermionic, while C is a real, bosonic spinor. The antifield spectrum is organized into the antifields $\{h^{*\mu\nu}, \psi^{*\mu}\}$ of the original fields $\{h_{\mu\nu}, \psi_\mu\}$ together with those of the ghosts $\{\eta^{*\mu}, \xi^*, C^*\}$, of statistics opposite to that of the associated fields/ghosts. The antifields $\psi^{*\mu}$ are spinor-vectors with real components and C^* is a purely imaginary spinor.

Since the gauge generators of the free theory are field independent, it follows that the BRST differential simply reduces to

$$s = \delta + \gamma, \tag{9}$$

where δ represents the Koszul-Tate differential, graded by the antighost number agh ($\mathrm{agh}(\delta) = -1$), and γ stands for the exterior derivative along the gauge orbits, whose degree is named pure ghost number pgh ($\mathrm{pgh}(\gamma) = 1$). These two degrees do not interfere ($\mathrm{pgh}(\delta) = 0$, $\mathrm{agh}(\gamma) = 0$). The overall degree that grades the BRST complex is known as the ghost number gh and is

defined like the difference between the pure ghost number and the antighost number, such that $\mathrm{gh}(s) = \mathrm{gh}(\delta) = \mathrm{gh}(\gamma) = 1$. If we denote by

$$\Phi^{\alpha_0} = (h_{\mu\nu}, \psi_\mu), \qquad \eta^{\alpha_1} = (\eta_\mu, \xi, C) \tag{10}$$

the fields and ghosts of the free theory and by

$$\Phi^*_{\alpha_0} = (h^{*\mu\nu}, \psi^{*\mu}), \qquad \eta^*_{\alpha_1} = (\eta^{*\mu}, \xi^*, C^*) \tag{11}$$

the corresponding antifields, then, according to the standard rules of the BRST formalism, the corresponding degrees of the generators from the BRST complex are valued like

$$\mathrm{agh}\left(\Phi^{\alpha_0}\right) = 0, \qquad \mathrm{agh}\left(\eta^{\alpha_1}\right) = 0, \tag{12}$$

$$\mathrm{agh}\left(\Phi^*_{\alpha_0}\right) = 1, \qquad \mathrm{agh}\left(\eta^*_{\alpha_1}\right) = 2, \tag{13}$$

$$\mathrm{pgh}\left(\Phi^{\alpha_0}\right) = 0, \qquad \mathrm{pgh}\left(\eta^{\alpha_1}\right) = 1, \tag{14}$$

$$\mathrm{pgh}\left(\Phi^*_{\alpha_0}\right) = 0, \qquad \mathrm{pgh}\left(\eta^*_{\alpha_1}\right) = 0. \tag{15}$$

The actions of the differentials δ and γ on the generators (10)–(11) from the BRST complex are given by

$$\delta h^{*\mu\nu} = 2\partial_\alpha \partial_\beta \mathcal{W}^{\mu\alpha\nu\beta}, \qquad \delta\psi^{*\mu} = -\varepsilon^{\mu\nu\rho\lambda}\partial_\nu \bar{\psi}_\rho \gamma_5 \gamma_\lambda, \tag{16}$$

$$\delta\eta^{*\mu} = -2\partial_\nu h^{*\mu\nu}, \qquad \delta\xi^* = 2h^*, \qquad \delta C^* = \partial_\mu \psi^{*\mu}, \tag{17}$$

$$\delta\Phi^{\alpha_0} = 0, \qquad \delta\eta^{\alpha_1} = 0, \tag{18}$$

$$\gamma\Phi^*_{\alpha_0} = 0, \qquad \gamma\eta^*_{\alpha_1} = 0, \tag{19}$$

$$\gamma h_{\mu\nu} = \partial_{(\mu}\eta_{\nu)} + 2\sigma_{\mu\nu}\xi, \qquad \gamma\psi_\mu = \partial_\mu C, \tag{20}$$

$$\gamma\eta_\mu = \gamma\xi = 0, \qquad \gamma C = 0. \tag{21}$$

The notation h^* signifies the trace of $h^{*\mu\nu}$, $h^* = \sigma_{\mu\nu} h^{*\mu\nu}$. The BRST differential is known to have a canonical action in a structure named antibracket and denoted by the symbol $(,)$ $(s\cdot = (\cdot, \bar{S}))$, which is obtained by decreeing the fields and ghosts respectively conjugated to the corresponding antifields. The generator of the BRST symmetry is a bosonic functional, of ghost number zero, which is solution to the classical master equation $\left(\bar{S}, \bar{S}\right) = 0$. In our case the solution to the master equation reads as

$$\bar{S} = S_0^L\left[h_{\mu\nu}, \psi_\mu\right] + \int d^4x \left[h^{*\mu\nu}\left(\partial_{(\mu}\eta_{\nu)} + 2\sigma_{\mu\nu}\xi\right) + \psi^{*\mu}\partial_\mu C\right]. \tag{22}$$

3 Deformation of the master equation: a brief review

We begin with a "free" gauge theory, described by a Lagrangian action $S_0^L\left[\Phi^{\alpha_0}\right]$, invariant under some gauge transformations

$$\delta_\epsilon \Phi^{\alpha_0} = Z^{\alpha_0}_{\ \ \alpha_1} \epsilon^{\alpha_1}, \qquad \frac{\delta S_0^L}{\delta \Phi^{\alpha_0}} Z^{\alpha_0}_{\ \ \alpha_1} = 0, \tag{23}$$

and consider the problem of constructing consistent interactions among the fields Φ^{α_0} such that the couplings preserve the field spectrum and the original number of gauge symmetries. This matter is addressed by means of reformulating the problem of constructing consistent interactions as a deformation problem of the solution to the master equation corresponding to the "free" theory [18]. Such a reformulation is possible due to the fact that the solution to the master equation contains all the information on the gauge structure of the theory. If an interacting gauge theory can be consistently constructed, then the solution $\bar{S}$ to the master equation associated with the "free" theory, $\left(\bar{S}, \bar{S}\right) = 0$, can be deformed into a solution S

$$\begin{aligned} \bar{S} \rightarrow S &= \bar{S} + gS_1 + g^2 S_2 + \cdots \\ &= \bar{S} + g \int d^D x \, a + g^2 \int d^D x \, b + \cdots \end{aligned} \tag{24}$$

of the master equation for the deformed theory

$$(S, S) = 0, \tag{25}$$

such that both the ghost and antifield spectra of the initial theory are preserved. The equation (25) splits, according to the various orders in the coupling constant (deformation parameter) g, into

$$\left(\bar{S}, \bar{S}\right) = 0, \tag{26}$$

$$2\left(S_1, \bar{S}\right) = 0, \tag{27}$$

$$2\left(S_2, \bar{S}\right) + (S_1, S_1) = 0, \tag{28}$$

$$\left(S_3, \bar{S}\right) + (S_1, S_2) = 0, \tag{29}$$

$$\vdots$$

The equation (26) is fulfilled by hypothesis. The next one requires that the first-order deformation of the solution to the master equation, S_1, is a

co-cycle of the "free" BRST differential $s\cdot = (\cdot, \bar{S})$. However, only cohomologically nontrivial solutions to (27) should be taken into account, as the BRST-exact ones can be eliminated by some (in general nonlinear) field redefinitions. This means that S_1 pertains to the ghost number zero cohomological space of s, $H^0(s)$, which is generically nonempty due to its isomorphism to the space of physical observables of the "free" theory. It has been shown (on behalf of the triviality of the antibracket map in the cohomology of the BRST differential) that there are no obstructions in finding solutions to the remaining equations ((28)–(29), etc.). However, the resulting interactions may be nonlocal, and there might even appear obstructions if one insists on their locality. The analysis of these obstructions can be done with the help of cohomological techniques.

4 Consistent interactions between the Weyl graviton and the massless Rarita-Schwinger field

4.1 Standard material: $H(\gamma)$ and $H(\delta|d)$

The aim of this paper is the investigation of the effective couplings that can be introduced between the Weyl graviton and the massless Rarita-Schwinger field. This matter is addressed in the context of the antifield-BRST deformation procedure described in the above and relies on computing the solutions to the equations (27)–(29), etc., with the help of the BRST cohomology. For obvious reasons, we consider only smooth, local, (background) Lorentz invariant and, moreover, Poincaré invariant quantities (i.e. we do not allow explicit dependence on the spacetime coordinates). If we make the notation $S_1 = \int d^4x\, a$, with a a local function, then the equation (27), which we have seen that controls the first-order deformation, takes the local form

$$sa = \partial_\mu m^\mu, \qquad \mathrm{gh}(a) = 0, \qquad \varepsilon(a) = 0, \tag{30}$$

for some local m^μ and it shows that the nonintegrated density of the first-order deformation pertains to the local cohomology of s in ghost number zero, $a \in H^0(s|d)$, where d denotes the exterior spacetime differential. The

solution to the equation (30) is unique up to s-exact pieces plus divergences

$$\begin{aligned} a &\rightarrow a+sb+\partial_\mu n^\mu, \\ \mathrm{gh}(b) &= -1, \qquad \varepsilon(b)=1, \qquad \mathrm{gh}\left(n^\mu\right)=0, \qquad \varepsilon\left(n^\mu\right)=0. \end{aligned} \tag{31}$$

At the same time, if the general solution of (30) is found to be completely trivial, $a=sb+\partial_\mu n^\mu$, then it can be made to vanish $a=0$.

In order to analyze the equation (30), we develop a according to the antighost number

$$a=\sum_{i=0}^{I} a_i, \qquad \mathrm{agh}(a_i)=i, \qquad \mathrm{gh}(a_i)=0, \qquad \varepsilon(a_i)=0, \tag{32}$$

and assume, without loss of generality, that the decomposition (32) stops at some finite value of I. This can be shown, for instance, like in [28] (Section 3), under the sole assumption that the interacting Lagrangian at the first order in the coupling constant, a_0, has a finite, but otherwise arbitrary derivative order. Replacing the decomposition (32) into the equation (30) and projecting it on various values of the antighost number, we obtain the tower of equations

$$\gamma a_I = \partial_\mu \overset{(I)^{\mu}}{m}, \tag{33}$$

$$\delta a_I + \gamma a_{I-1} = \partial_\mu \overset{(I-1)^{\mu}}{m}, \tag{34}$$

$$\delta a_i + \gamma a_{i-1} = \partial_\mu \overset{(i-1)^{\mu}}{m}, \qquad (1 \leq i \leq I-1), \tag{35}$$

where $\left(\overset{(i)^{\mu}}{m}\right)_{i=\overline{0,I}}$ are some local currents with $\mathrm{agh}\left(\overset{(i)^{\mu}}{m}\right)=i$. It can be proved that the equation (33) can be replaced in strictly positive antighost numbers by

$$\gamma a_I = 0, \qquad I>0. \tag{36}$$

The proof of this result is standard material and can be found for instance in [25, 26, 28, 29, 30, 31]. Due to the second-order nilpotency of γ ($\gamma^2=0$), the solution to the equation (36) is clearly unique up to γ-exact contributions

$$\begin{aligned} &a_I \rightarrow a_I+\gamma b_I, \\ &\mathrm{agh}(b_I)=I, \qquad \mathrm{pgh}(b_I)=I-1, \qquad \varepsilon(b_I)=1. \end{aligned} \tag{37}$$

Meanwhile, if it turns out that a_I reduces to γ-exact terms only, $a_I = \gamma b_I$, then it can be made to vanish, $a_I = 0$. In other words, the nontriviality of the first-order deformation a is translated at its highest antighost number component into the requirement that $a_I \in H^I(\gamma)$, where $H^I(\gamma)$ denotes the cohomology of the exterior longitudinal derivative γ in pure ghost number equal to I. So, in order to solve the equation (30) (equivalent with (36) and (34)–(35)), we need to compute the cohomology of γ, $H(\gamma)$, and, as it will be made clear below, also the local cohomology of δ, $H(\delta|d)$.

In order to determine the cohomology $H(\gamma)$, we split the differential γ into two pieces

$$\gamma = \gamma_W + \gamma_{\mathrm{RS}}, \tag{38}$$

where γ_W acts nontrivially only on the fields/ghosts from the Weyl sector, while γ_{RS} does the same thing, but with respect to the Rarita-Schwinger sector. From the above splitting it follows that the nilpotency of γ is equivalent to the nilpotency and anticommutation of its components

$$(\gamma_W)^2 = 0 = (\gamma_{\mathrm{RS}})^2, \qquad \gamma_W \gamma_{\mathrm{RS}} + \gamma_{\mathrm{RS}} \gamma_W = 0, \tag{39}$$

so finally we find the isomorphism

$$H(\gamma) = H(\gamma_W) \otimes H(\gamma_{\mathrm{RS}}). \tag{40}$$

Using the results from the literature concerning the cohomologies $H(\gamma_W)$ and $H(\gamma_{\mathrm{RS}})$ [25, 26] we can state that $H(\gamma)$ is generated on the one hand by $\Phi^*_{\alpha_0}$, $\eta^*_{\alpha_1}$, $\partial_{[\mu}\psi_{\nu]}$ and $\mathcal{W}_{\mu\nu\alpha\beta}$ as well as by their spacetime derivatives and, on the other hand, by the ghosts C, η_μ, $\partial_{[\mu}\eta_{\nu]}$, ξ and $\partial_\mu \xi$. So, the most general (and nontrivial), local solution to (36) can be written, up to γ-exact contributions, as

$$a_I = \alpha_I \left(\left[\partial_{[\mu}\psi_{\nu]}\right], \left[\mathcal{W}_{\mu\nu\alpha\beta}\right], \left[\Phi^*_{\alpha_0}\right], \left[\eta^*_{\alpha_1}\right] \right) \omega^I \left(C, \eta_\mu, \partial_{[\mu}\eta_{\nu]}, \xi, \partial_\mu \xi \right), \tag{41}$$

where the notation $f([q])$ means that f depends on q and its derivatives up to a finite order, while ω^I denotes the elements of a basis in the space of polynomials with pure ghost number I in the corresponding ghosts and some of their first-order derivatives. The objects α_I (obviously nontrivial in $H^0(\gamma)$) were taken to have a bounded number of derivatives, and therefore they are polynomials in the antifields $\Phi^*_{\alpha_0}$ and $\eta^*_{\alpha_1}$, in the linearized Weyl tensor $\mathcal{W}_{\mu\nu\alpha\beta}$, in the objects $\partial_{[\mu}\psi_{\nu]}$, as well as in their derivatives. They are required to fulfill the property $\mathrm{agh}(\alpha_I) = I$ in order to ensure that the ghost

number of a_I is equal to zero. Due to their γ-closeness, $\gamma\alpha_I = 0$, α_I will be called "invariant polynomials". In zero antighost number, the invariant polynomials are polynomials in the linearized Weyl tensor $\mathcal{W}_{\mu\nu\alpha\beta}$, in the objects $\partial_{[\mu}\psi_{\nu]}$, and also in their derivatives.

Substituting (41) in (34) we obtain that a necessary (but not sufficient) condition for the existence of (nontrivial) solutions a_{I-1} is that the invariant polynomials α_I are (nontrivial) objects from the local cohomology of Koszul-Tate differential $H(\delta|d)$ in antighost number $I>0$ and pure ghost number zero,

$$\begin{aligned} &\delta\alpha_I = \partial_\mu \overset{(I-1)^\mu}{j}, \\ &\mathrm{agh}\left(\overset{(I-1)^\mu}{j}\right) = I-1, \qquad \mathrm{pgh}\left(\overset{(I-1)^\mu}{j}\right) = 0. \end{aligned} \tag{42}$$

We recall that the local cohomology $H(\delta|d)$ is completely trivial in both strictly positive antighost *and* pure ghost numbers (for instance, see [32, 33], Theorem 5.4 and [34]). Using the fact that the Cauchy order of the free theory under study is equal to two together with the general results from [32, 33], according to which the local cohomology of the Koszul-Tate differential in pure ghost number zero is trivial in antighost numbers strictly greater than its Cauchy order, we can state that

$$H_J(\delta|d) = 0 \qquad \text{for all} \qquad J>2, \tag{43}$$

where $H_J(\delta|d)$ denotes the local cohomology of the Koszul-Tate differential in antighost number J and in zero pure ghost number. It is quite reasonable to assume that if the invariant polynomial α_J, with $\mathrm{agh}(\alpha_J) = J \geq 2$, is trivial in $H_J(\delta|d)$, then it can be taken to be trivial also in $H_J^{\mathrm{inv}}(\delta|d)$

$$\left(\alpha_J = \delta b_{J+1} + \partial_\mu \overset{(J)^\mu}{c}, \qquad \mathrm{agh}\left(\alpha_J\right) = J \geq 2\right) \Rightarrow \left(\alpha_J = \delta\beta_{J+1} + \partial_\mu \overset{(J)^\mu}{\gamma}\right), \tag{44}$$

with both β_{J+1} and $\overset{(J)^\mu}{\gamma}$ invariant polynomials. Here, $H_J^{\mathrm{inv}}(\delta|d)$ denotes the invariant characteristic cohomology in antighost number J (the local cohomology of the Koszul-Tate differential in the space of invariant polynomials). This assumption is based on what happens in many gauge theories [25, 26, 28, 29, 30, 31, 35]. The results (43)–(44) yield the conclusion that

$$H_J^{\mathrm{inv}}(\delta|d) = 0, \qquad \text{for all} \qquad J>2. \tag{45}$$

With the help of the definitions (16)–(18) we observe that both $H_2(\delta|d)$ in pure ghost number zero and $H_2^{\mathrm{inv}}(\delta|d)$ are spanned only by the undifferentiated antifields

$$H_2(\delta|d) \text{ and } H_2^{\mathrm{inv}}(\delta|d): \qquad (C^*, \eta^{*\mu})\,. \tag{46}$$

In contrast to the groups $(H_J(\delta|d))_{J\geq 2}$ and $\left(H_J^{\mathrm{inv}}(\delta|d)\right)_{J\geq 2}$, which are finite-dimensional, the cohomology $H_1(\delta|d)$ in pure ghost number zero, that is related to global symmetries and ordinary conservation laws, is infinite-dimensional since the theory is free. Fortunately, it will not be needed in the sequel.

The previous results on $H(\delta|d)$ and $H^{\mathrm{inv}}(\delta|d)$ in strictly positive antighost numbers are important because they control the obstructions to removing the antifields from the first-order deformation. This statement is also standard material and can be done like in [25, 26, 28, 29, 30, 31, 35]. Its proof is mainly based on the formulas (43)–(45) and relies on the fact that we can successively eliminate all the pieces of antighost number strictly greater than two from the nonintegrated density of the first-order deformation by adding only trivial terms, so we can take, without loss of nontrivial objects, the condition $I \leq 2$ in the decomposition (32). In addition, the last representative is of the form (41), where the invariant polynomial is necessarily a nontrivial object from $H_2^{\mathrm{inv}}(\delta|d)$ for $I = 2$, respectively from $H_1(\delta|d)$ for $I = 1$.

4.2 Case $I = 2$

In the case $I = 2$ the nonintegrated density of the first-order deformation (32) becomes

$$a = a_0 + a_1 + a_2. \tag{47}$$

We can further decompose a in a natural manner as

$$a = a^{(W)} + a^{(\mathrm{RS})} + a^{(\mathrm{int})}, \tag{48}$$

where $a^{(W)}$ contains only fields/ghosts/antifields from the Weyl sector, $a^{(\mathrm{RS})}$ is strictly related to the Rarita-Schwinger theory and $a^{(\mathrm{int})}$ describes the cross-interactions between the two theories, so it effectively mixes both sectors. Each of the three components satisfies an individual equation of the type (30) and admits a decomposition similar to (47)

$$a_k = a_k^{(W)} + a_k^{(\mathrm{RS})} + a_k^{(\mathrm{int})}, \qquad (k = 0, 1, 2), \tag{49}$$

so each type of deformation is subject to a set of equations of the form (36) and (34)–(35) for $I = 2$. We are interested only in the term $a_k^{(\mathrm{int})}$ since the others merely describe the self-interactions of Weyl and respectively of massless Rarita-Schwinger theory, which have already been studied in the literature. The self-interactions of Weyl theory are known to describe the Weyl gravity action [25], while the Rarita-Schwinger model leads to no consistent self-interactions [26]. Using the formula (41) for pure ghost number two and the result (46), we deduce that the most general, nontrivial element $a_2^{(\mathrm{int})}$ (as solution to the equation $\gamma a_2^{(\mathrm{int})} = 0$) is

$$\begin{aligned} a_2^{(\mathrm{int})} &= k_1 \eta^{*\mu} \bar{C} \gamma_\mu C + k_2 \partial_{[\mu} \eta_{\nu]} C^* \gamma^{[\mu} \gamma^{\nu]} C + k_3 \eta_\mu C^* \gamma^\mu C \\ &\quad + k_4 \left(\partial_\mu \xi\right) C^* \gamma^\mu C + k_5 \xi C^* C, \end{aligned} \tag{50}$$

with $(k_a)_{a=\overline{1,5}}$ some arbitrary complex constants (k_2 and k_5 are real numbers and the others are purely imaginary). On behalf of the definitions (16)–(21), from (50) we get

$$\begin{aligned} \delta a_2^{(\mathrm{int})} &= \partial_\mu j_1^\mu + \gamma b_1 \\ &\quad - \tfrac{1}{2} k_3 \partial_{[\mu} \eta_{\nu]} \psi^{*\mu} \gamma^\nu C + k_3 \xi \psi^{*\mu} \gamma_\mu C \\ &\quad - \partial_\nu \xi \left(4 k_2 \psi_\mu^* \gamma^{[\mu} \gamma^{\nu]} C + k_5 \psi^{*\nu} C\right), \end{aligned} \tag{51}$$

where

$$\begin{aligned} j_1^\mu &= -2 k_1 h^{*\mu\nu} \bar{C} \gamma_\nu C + k_2 \partial_{[\alpha} \eta_{\beta]} \psi^{*\mu} \gamma^{[\alpha} \gamma^{\beta]} C \\ &\quad + \left(k_4 \left(\partial_\alpha \xi\right) + k_3 \eta_\alpha\right) \psi^{*\mu} \gamma^\alpha C + k_5 \xi \psi^{*\mu} C, \end{aligned} \tag{52}$$

$$\begin{aligned} b_1 &= 4 k_1 h^{*\mu\nu} \bar{C} \gamma_\mu \psi_\nu - k_2 \partial_{[\mu} \eta_{\nu]} \psi_\rho^* \gamma^{[\mu} \gamma^{\nu]} \psi^\rho \\ &\quad - \left[k_3 \eta_\mu + k_4 \left(\partial_\mu \xi\right)\right] \psi_\rho^* \gamma^\mu \psi^\rho \\ &\quad - k_5 \xi \psi_\rho^* \psi^\rho + \left(k_4 \mathcal{K}_{\mu\nu} - \tfrac{1}{2} k_3 h_{\mu\nu}\right) \psi^{*\mu} \gamma^\nu C \\ &\quad - k_2 \partial_{[\mu} h_{\nu]\rho} \psi^{*\rho} \gamma^{[\mu} \gamma^{\nu]} C. \end{aligned} \tag{53}$$

By means of the relation (51), we observe that the existence of $a_1^{(\mathrm{int})}$ as solution to the equation $\delta a_2^{(\mathrm{int})} + \gamma a_1^{(\mathrm{int})} = \partial_\mu \overset{(1)}{m}{}^{(\mathrm{int})\mu}$ requires that

$$k_2 = k_3 = k_5 = 0. \tag{54}$$

Inserting (54) in (50) and (51), we have that the pieces of antighost number two and one from $a^{(\mathrm{int})}$ respectively read as

$$a_2^{(\mathrm{int})} = k_1 \eta^{*\mu} \bar{C} \gamma_\mu C + k_4 \left(\partial_\mu \xi\right) C^* \gamma^\mu C, \tag{55}$$

$$a_1^{(\mathrm{int})} = -4k_1 h^{*\mu\nu} \bar{C} \gamma_\mu \psi_\nu + k_4 \left[\left(\partial_\mu \xi\right) \psi_\rho^* \gamma^\mu \psi^\rho - \mathcal{K}_{\mu\nu} \psi^{*\mu} \gamma^\nu C\right]. \tag{56}$$

After some computation we find that

$$\delta a_1^{(\mathrm{int})} = \partial_\mu j_0^\mu + \gamma b_0 + c_0, \tag{57}$$

where

$$\begin{aligned} j_0^\mu \; = \; & -8k_1 \left(\partial_\beta \mathcal{W}^{\mu\alpha\nu\beta}\right) \bar{\psi}_\alpha \gamma_\nu C + k_4 \varepsilon^{\mu\nu\rho\lambda} \left[\tfrac{1}{2} \left(\partial_\alpha \xi\right) \bar{\psi}_\nu \gamma_5 \gamma_\rho \gamma^\alpha \psi_\lambda \right. \\ & \left. + \xi \left(\partial_\nu \bar{\psi}_\rho\right) \gamma_5 \psi_\lambda - \mathcal{K}_{\nu\alpha} \bar{\psi}_\rho \gamma_5 \gamma_\lambda \gamma^\alpha C\right], \end{aligned} \tag{58}$$

$$b_0 = -4k_1 \left(\partial_\beta \mathcal{W}^{\mu\alpha\nu\beta}\right) \bar{\psi}_\mu \gamma_\nu \psi_\alpha - \tfrac{1}{2} k_4 \mathcal{K}_{\mu\alpha} \varepsilon^{\mu\nu\rho\lambda} \bar{\psi}_\nu \gamma_5 \gamma_\rho \gamma^\alpha \psi_\lambda, \tag{59}$$

$$\begin{aligned} c_0 \; = \; & 4k_1 \left(\partial_\beta \mathcal{W}^{\mu\alpha\nu\beta}\right) \partial_{[\mu} \bar{\psi}_{\alpha]} \gamma_\nu C \\ & + k_4 \varepsilon^{\mu\nu\rho\lambda} \left[\left(\partial_\mu \mathcal{K}_{\nu\alpha}\right) \bar{\psi}_\rho \gamma_5 \gamma_\lambda \gamma^\alpha C + \xi \left(\partial_\mu \bar{\psi}_\nu\right) \gamma_5 \left(\partial_\rho \psi_\lambda\right)\right]. \end{aligned} \tag{60}$$

Combining (57) with (60), we observe that the existence of $a_0^{(\mathrm{int})}$ as solution to the equation $\delta a_1^{(\mathrm{int})} + \gamma a_0^{(\mathrm{int})} = \partial_\mu \overset{(0)}{m}{}^{(\mathrm{int})\mu}$ implies that $c_0 = 0$, which is equivalent to the vanishing of the remaining constants

$$k_1 = k_4 = 0. \tag{61}$$

Replacing (61) in (55)–(56), we conclude there is no nontrivial first-order cross-coupling that stops at antighost number two. It is remarkable to note that the impossibility of consistent, nontrivial, first-order deformations that end at antighost number two was obtained *without* imposing any restriction on the derivative order of the interacting Lagrangian.

4.3 Case $I = 1$

The next step is to investigate whether there exist nontrivial, cross-coupling first-order deformations that end at antighost number one ($I = 1$)

$$a^{(\mathrm{int})} = a_0^{(\mathrm{int})} + a_1^{(\mathrm{int})}, \tag{62}$$

with $a_1^{(\mathrm{int})}$ from $H^1(\gamma)$. According to (41), we deduce that the most general form of $a_1^{(\mathrm{int})}$ that might provide effective cross-interactions is written like

$$a_1^{(\mathrm{int})} = \psi^{*\mu}\left(\tilde{M}_{\mu\nu}\partial^\nu\xi + \bar{M}_\mu{}^{\alpha\beta}\partial_{[\alpha}\eta_{\beta]} + \hat{M}_\mu\xi + \hat{M}_{\mu\nu}\eta^\nu\right) + h^{*\mu\nu}\bar{C}M_{\mu\nu}, \quad (63)$$

where $\tilde{M}_{\mu\nu}$, $\bar{M}_\mu{}^{\alpha\beta}$, $\hat{M}_\mu$, $\hat{M}_{\mu\nu}$ and $M_{\mu\nu}$ are spinor-tensors depending on the original fields, which are gauge-invariant, fermionic objects (each of them can be generically written like $M_\Delta = U_\Delta^{\alpha\beta}\partial_{[\alpha}\psi_{\beta]}$, where $U_\Delta^{\alpha\beta}$ are bosonic matrices, possibly involving $\mathcal{W}^{\mu\alpha\nu\beta}$). All the matrices appearing in (63) must be purely imaginary. Direct calculations based on the definitions (16)–(21) lead to

$$\delta a_1^{(\mathrm{int})} = \partial_\mu m_0^\mu + \gamma e_0 + f_0, \quad (64)$$

where we made the notations

$$\begin{aligned} m_0^\mu = {} & \varepsilon^{\mu\nu\rho\lambda}\bar{\psi}_\nu\gamma_5\gamma_\rho\left(\tilde{M}_{\lambda\alpha}\partial^\alpha\xi + \bar{M}_\lambda{}^{\alpha\beta}\partial_{[\alpha}\eta_{\beta]} + \hat{M}_\lambda\xi + \hat{M}_{\lambda\alpha}\eta^\alpha\right) \\ & +2\left(\partial_\beta\mathcal{W}^{\mu\alpha\nu\beta}\right)\bar{C}M_{\alpha\nu} + \mathcal{W}^{\mu\alpha\nu\beta}\bar{C}\partial_{[\nu}M_{\beta]\alpha} \\ & +\varepsilon^{\mu\nu\rho\lambda}\bar{C}\gamma_5\gamma_\nu\left(-\tilde{M}_{\rho\alpha}\mathcal{K}_\lambda^\alpha + \bar{M}_\rho{}^{\alpha\beta}\partial_{[\alpha}h_{\beta]\lambda} + \tfrac{1}{2}\hat{M}_{\rho\alpha}h_\lambda^\alpha\right), \end{aligned} \quad (65)$$

$$\begin{aligned} e_0 = {} & \varepsilon^{\mu\nu\rho\lambda}\bar{\psi}_\mu\gamma_5\gamma_\nu\left(-\tilde{M}_{\rho\alpha}\mathcal{K}_\lambda^\alpha + \bar{M}_\rho{}^{\alpha\beta}\partial_{[\alpha}h_{\beta]\lambda} + \tfrac{1}{2}\hat{M}_{\rho\alpha}h_\lambda^\alpha\right) \\ & -2\left(\partial_\beta\mathcal{W}^{\mu\alpha\nu\beta}\right)\bar{\psi}_\alpha M_{\mu\nu} - \mathcal{W}^{\mu\alpha\nu\beta}\bar{\psi}_\beta\partial_{[\mu}M_{\alpha]\nu}, \end{aligned} \quad (66)$$

$$\begin{aligned} f_0 = {} & -\tfrac{1}{2}\varepsilon^{\mu\nu\rho\lambda}\bar{\psi}_\mu\gamma_5\gamma_\nu\left[\partial_{[\rho}\hat{M}_{\lambda]\alpha}\eta^\alpha + \left(\partial_{[\rho}\bar{M}_{\lambda]}{}^{\alpha\beta} + \tfrac{1}{4}\hat{M}_{[\rho}{}^{[\alpha}\delta_{\lambda]}^{\beta]}\right)\partial_{[\alpha}\eta_{\beta]}\right. \\ & \left. + \left(\partial_{[\rho}\hat{M}_{\lambda]} + \hat{M}_{[\mu\nu]}\right)\xi + \left(-\hat{M}_{[\rho}\delta_{\lambda]}^\alpha + \partial_{[\rho}\tilde{M}_{\lambda]}{}^\alpha - 4\bar{M}_{[\rho\lambda]}{}^\alpha\right)\partial_\alpha\xi\right] \\ & +\bar{C}\left[\varepsilon^{\mu\nu\rho\lambda}\gamma_5\gamma_\mu\partial_\nu\left(-\tilde{M}_{\rho\alpha}\mathcal{K}_\lambda^\alpha + \bar{M}_\rho{}^{\alpha\beta}\partial_{[\alpha}h_{\beta]\lambda} + \tfrac{1}{2}\hat{M}_{\rho\alpha}h_\lambda^\alpha\right)\right. \\ & \left. +\tfrac{1}{2}\mathcal{W}^{\mu\alpha\nu\beta}\partial_{[\mu}M_{\alpha][\nu,\beta]}\right]. \end{aligned} \quad (67)$$

If we look at (64), we observe that the existence of $a_0^{(\mathrm{int})}$ as solution to the equation $\delta a_1^{(\mathrm{int})} + \gamma a_0^{(\mathrm{int})} = \partial_\mu \overset{(0)}{m}{}^{(\mathrm{int})\mu}$ requires that

$$f_0 = 0. \quad (68)$$

This observation leads to the following identities that must be satisfied by the various fermionic spinor-tensors

$$\partial_{[\rho}\hat{M}_{\lambda]\alpha} = 0, \tag{69}$$

$$\partial_{[\rho}\bar{M}_{\lambda]}{}^{\alpha\beta} + \tfrac{1}{4}\hat{M}_{[\rho}{}^{[\alpha}\delta_{\lambda]}^{\beta]} = 0, \tag{70}$$

$$\partial_{[\rho}\hat{M}_{\lambda]} + \hat{M}_{[\mu\nu]} = 0, \tag{71}$$

$$-\hat{M}_{[\rho}\delta_{\lambda]}^{\alpha} + \partial_{[\rho}\tilde{M}_{\lambda]}{}^{\alpha} - 4\bar{M}_{[\rho\lambda]}^{\ \ \alpha} = 0, \tag{72}$$

$$\begin{aligned}&\varepsilon^{\mu\nu\rho\lambda}\gamma_5\gamma_\mu\partial_\nu\left(-\tilde{M}_{\rho\alpha}\mathcal{K}_\lambda^\alpha + \bar{M}_\rho{}^{\alpha\beta}\partial_{[\alpha}h_{\beta]\lambda} + \tfrac{1}{2}\hat{M}_{\rho\alpha}h_\lambda^\alpha\right)\\&+\tfrac{1}{2}\mathcal{W}^{\mu\alpha\nu\beta}\partial_{[\mu}M_{\alpha][\nu,\beta]} = 0.\end{aligned} \tag{73}$$

The solutions to the equations (69)–(72) can be expressed in terms of some arbitrary, fermionic, spinor and gauge-invariant objects, like

$$\begin{aligned}\hat{M}_{\mu\alpha} &= \partial_\mu\hat{N}_\alpha, && (74)\\ \hat{M}_\alpha &= -\hat{N}_\alpha + \partial_\alpha\hat{M}, && (75)\\ \bar{M}_\mu{}^{\alpha\beta} &= -\tfrac{1}{4}\hat{N}^{[\alpha}\delta_\mu^{\beta]} + \partial_\mu\bar{M}^{\alpha\beta}, && (76)\\ \tilde{M}_{\mu\nu} &= 4\bar{M}_{\mu\nu} + \hat{M}\sigma_{\mu\nu} + \partial_\mu\tilde{M}_\alpha, && (77)\end{aligned}$$

where $\bar{M}_{\mu\nu}$ is antisymmetric. Using the solutions (74)–(77) in (73), we then find

$$\begin{aligned}&\tfrac{1}{2}\mathcal{W}^{\mu\nu\alpha\beta}\left[\partial_{[\mu}M_{\nu][\alpha,\beta]} + \gamma_5\gamma^\rho\partial^\lambda\left(\varepsilon_{\mu\nu\rho\lambda}\bar{M}_{\alpha\beta} + \varepsilon_{\alpha\beta\rho\lambda}\bar{M}_{\mu\nu}\right)\right]\\&+\tfrac{1}{2}\varepsilon^{\mu\nu\rho\lambda}\gamma_5\gamma_\mu\left(4\bar{M}_{\nu\alpha} + \partial_\nu\tilde{M}_\alpha\right)\partial_{[\rho}\mathcal{K}_{\lambda]}{}^{\alpha} = 0.\end{aligned} \tag{78}$$

This last equality yields

$$\partial_{[\mu}M_{\nu][\alpha,\beta]} + \gamma_5\gamma^\rho\partial^\lambda\left(\varepsilon_{\mu\nu\rho\lambda}\bar{M}_{\alpha\beta} + \varepsilon_{\alpha\beta\rho\lambda}\bar{M}_{\mu\nu}\right) = 0, \tag{79}$$

$$4\bar{M}_{\nu\alpha} + \partial_\nu\tilde{M}_\alpha = 0. \tag{80}$$

If we take the symmetric part of (80) and perform some simple computation, we get

$$\tilde{M}_\alpha = 0, \tag{81}$$

which then produces

$$\bar{M}_{\nu\alpha} = 0. \tag{82}$$

Replacing the results (81)–(82) into (79), we obtain the equation

$$\partial_{[\mu} M_{\nu][\alpha,\beta]} = 0. \tag{83}$$

Putting together the relations (74)–(77) and (81)–(83), we conclude that the solutions to the equations (69)–(73) can be written like

$$\hat{M}_{\mu\alpha} = \partial_\mu \hat{N}_\alpha, \tag{84}$$

$$\hat{M}_\alpha = -\hat{N}_\alpha + \partial_\alpha \hat{M}, \tag{85}$$

$$\bar{M}_\mu{}^{\alpha\beta} = -\tfrac{1}{4} \hat{N}^{[\alpha} \delta_\mu^{\beta]}, \tag{86}$$

$$\tilde{M}_{\mu\nu} = \hat{M} \sigma_{\mu\nu}, \tag{87}$$

$$M_{\mu\nu} = \partial_{(\mu} M_{\nu)}, \tag{88}$$

in terms of some arbitrary, fermionic, spinor, gauge-invariant objects $\hat{N}_\alpha$, $\hat{M}$ and M_ν. Inserting the solutions (84)–(88) into (63), after some computation we arrive at

$$a_1^{(\mathrm{int})} = \partial_\mu n^\mu + sq + \gamma r, \tag{89}$$

where the following notations were employed

$$n^\mu = \psi^{*\mu} \left(\hat{M}\xi + \hat{N}^\nu \eta_\nu \right) + 2h^{*\mu\nu} \bar{C} M_\nu, \tag{90}$$

$$r = -\tfrac{1}{2} \psi^{*\mu} \hat{N}^\nu h_{\mu\nu} + 2h^{*\mu\nu} \bar{\psi}_\mu M_\nu, \tag{91}$$

$$q = -C^* \left(\hat{M}\xi + \hat{N}^\nu \eta_\nu \right) - 2\eta^{*\mu} \bar{C} M_\mu. \tag{92}$$

As we have previously discussed, the solution to $a_1^{(\mathrm{int})}$ is unique on the one hand up to γ-trivial contributions (since it represents the component of highest antighost number from the first-order deformation, as it can be seen from the expansion (62)) and, on the other hand, up to s-exact modulo d terms (since it belongs to the first-order deformation), so we can choose

$$a_1^{(\mathrm{int})} = 0.$$

In conclusion, there are no nontrivial, cross-coupling first-order deformations that end at antighost number one. It is important to note that the absence of consistent, nontrivial first-order deformations that end at antighost number one also emerged *without* imposing any restriction on the derivative order of the interacting Lagrangian.

4.4 Case $I = 0$

At this stage, we are left with a sole possibility, namely, that $a^{(\mathrm{int})}$ reduces to its antighost number zero component ($I = 0$)

$$a^{(\mathrm{int})} = a_0^{(\mathrm{int})}, \tag{93}$$

which is subject to the equation

$$\gamma a_0^{(\mathrm{int})} = \partial_\mu \overset{(0)}{m}{}^{(\mathrm{int})\mu}, \tag{94}$$

for some local $\overset{(0)}{m}{}^{(\mathrm{int})\mu}$. There are two main types of solutions to (94). The first type, to be denoted by $a_0'^{(\mathrm{int})}$, corresponds to $\overset{(0)}{m}{}^{(\mathrm{int})\mu} = 0$ and is given by gauge-invariant, nonintegrated densities constructed from the original fields and their spacetime derivatives, which, according to (41), are given by

$$a_0'^{(\mathrm{int})} = a_0'^{(\mathrm{int})} \left(\left[\partial_{[\mu} \psi_{\nu]} \right], \left[\mathcal{W}_{\mu\nu\alpha\beta} \right] \right), \tag{95}$$

up to the condition that they effectively describe cross-couplings between the two types of fields and cannot be written in a divergence-like form. At this point we invoke the hypothesis on the preservation of the number of derivatives on each field, which means here that the following two requirements are simultaneously satisfied: (i) the derivative order of the equations of motion on each field is the same for the free and respectively for the interacting theory; (ii) the maximum number of derivatives in the interaction vertices is equal to four, i.e. the maximum number of derivatives from the free lagrangian. This further yields the trivial solution

$$a_0'^{(\mathrm{int})} = 0. \tag{96}$$

If we however relax the derivative-order condition, we can find nonvanishing solutions of the type (95)[1]. In conclusion, the condition on the number of derivatives prevents the appearance of the solutions of the first type.

The second kind of solutions, to be denoted by $a_0''^{(\mathrm{int})}$, is associated with $\overset{(0)}{m}{}^{(\mathrm{int})\mu} \neq 0$ in (94)

$$\gamma a_0''^{(\mathrm{int})} = \partial_\mu \overset{(0)}{m}{}^{(\mathrm{int})\mu}. \tag{97}$$

[1] An example of a possible solution of the form (95) is represented by the cubic vertex $a_0^{(\mathrm{int})} = \mathcal{W}^{\mu\nu\alpha\beta} \left(\partial_{[\mu} \bar{\psi}_{\nu]} \right) \gamma_5 \partial_{[\alpha} \psi_{\beta]}$.

In order to solve the equation (97) we recall the requirement that $a_0^{\prime\prime(\mathrm{int})}$ may contain at most four derivatives of the fields. Thus, it is useful to decompose $a_0^{\prime\prime(\mathrm{int})}$ according to the number of derivatives into

$$a_0^{\prime\prime(\mathrm{int})} = \lambda_0 + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4, \tag{98}$$

where $(\lambda_i)_{i=\overline{0,4}}$ contains i derivatives. Substituting (98) in (97), it follows that the latter equation becomes equivalent to two independent equations, one for each derivative order

$$\begin{aligned} \gamma\lambda_0 &= \partial_\mu p^\mu, & (99)\\ \gamma\lambda_1 &= \partial_\mu q^\mu, & (100)\\ \gamma\lambda_k &= \partial_\mu l_k^\mu,\ k=2,3,4. & (101) \end{aligned}$$

As λ_0 has no derivatives, we find that

$$\gamma\lambda_0 = \frac{\partial^R\lambda_0}{\partial\psi_\mu}\partial_\mu C + \frac{\partial\lambda_0}{\partial h_{\mu\nu}}\left(\partial_{(\mu}\eta_{\nu)} + 2\sigma_{\mu\nu}\xi\right). \tag{102}$$

The right-hand side of (102) can be written like in the right-hand side of (99) if

$$\partial_\mu\left(\frac{\partial^R\lambda_0}{\partial\psi_\mu}\right) = 0, \qquad \partial_\mu\left(\frac{\partial\lambda_0}{\partial h_{\mu\nu}}\right) = 0, \qquad \frac{\partial\lambda_0}{\partial h_{\mu\nu}}\sigma_{\mu\nu} = 0. \tag{103}$$

It is easy to see that the only solution to the all of the above equations is a (real) constant, which can always be taken to vanish. Let us analyze now the equation (100). We denote the Euler-Lagrange derivatives of λ_1 by

$$\bar{A}^{\alpha\beta} = \frac{\delta\lambda_1}{\delta h_{\alpha\beta}}, \qquad \bar{B}^\mu = \frac{\delta^R\lambda_1}{\delta\psi_\mu}, \tag{104}$$

and ask that (the symmetric tensor) $\bar{A}^{\alpha\beta}$ contains at least two massless Rarita-Schwinger fields and that $\bar{B}^\mu$ includes at least one Weyl graviton (in order to enforce the existence of effective cross-couplings). At the same time, $\bar{A}^{\alpha\beta}$ and $\bar{B}^\mu$ are precisely of order one in the field derivatives, with both $\bar{A}^{\alpha\beta}$ and $\bar{B}^\mu$ real objects (the first is bosonic and the second fermionic), and, moreover, $\bar{B}^\mu$ is a spinor-vector. Using the definitions (20), it follows that the equation (100) further restricts $\bar{A}^{\alpha\beta}$ and $\bar{B}^\mu$ to satisfy the equations

$$\partial_\mu\bar{B}^\mu = 0, \tag{105}$$

$$\partial_\alpha \bar{A}^{\alpha\beta} = 0, \qquad \bar{A}^{\alpha\beta}\sigma_{\alpha\beta} = 0. \tag{106}$$

The solutions to the above equations are

$$\begin{aligned} \bar{B}^\mu &= \partial_\nu \bar{B}^{\nu\mu}, \qquad \bar{B}^{\mu\nu} = -\bar{B}^{\nu\mu}, &(107)\\ \bar{A}^{\alpha\beta} &= \partial_\rho \bar{A}^{\rho\alpha\beta}, \qquad \bar{A}^{\rho\alpha\beta} = -\bar{A}^{\alpha\rho\beta}, \qquad \bar{A}^{\rho\alpha\beta}\sigma_{\alpha\beta} = 0, &(108)\end{aligned}$$

where the antisymmetric tensors $\bar{B}^{\mu\nu}$ and $\bar{A}^{\rho\alpha\beta}$ depend only on the original undifferentiated fields. The tensors $\bar{B}^{\mu\nu}$ and $\bar{A}^{\rho\alpha\beta}$ have the same properties (Grassmann parity, reality, spinor-like behavior) like $\bar{B}^\mu$ and respectively $\bar{A}^{\alpha\beta}$. We insist on the fact that a solution of the type $\bar{A}^{\mu\nu} = \partial_\alpha\partial_\beta D^{\mu\alpha\nu\beta}$, with $D^{\mu\alpha\nu\beta}$ possessing the symmetry properties of the Riemann tensor, is not allowed in our case due to the hypothesis on the derivative order, and hence (108) is the most general solution to the equations (106) in this case.

Let N be a derivation in the algebra of the fields and of their spacetime derivatives, that counts the number of the fields and of their derivatives, defined by

$$\begin{aligned} N &= \sum_{k\geq 0}\left[\frac{\partial^R}{\partial\left(\partial_{\mu_1}\cdots\partial_{\mu_k}\psi_\alpha\right)}\left(\partial_{\mu_1}\cdots\partial_{\mu_k}\psi_\alpha\right)\right.\\ &\quad \left. + \left(\partial_{\mu_1}\cdots\partial_{\mu_k}h_{\alpha\beta}\right)\frac{\partial}{\partial\left(\partial_{\mu_1}\cdots\partial_{\mu_k}h_{\alpha\beta}\right)}\right]. \qquad (109)\end{aligned}$$

Then, it is easy to see that for every nonintegrated density u we have that

$$Nu = \frac{\delta^R u}{\delta\psi_\alpha}\psi_\alpha + \frac{\delta u}{\delta h_{\alpha\beta}}h_{\alpha\beta} + \partial_\mu s^\mu, \tag{110}$$

where $\delta^R u/\delta\psi_\alpha$ and $\delta u/\delta h_{\alpha\beta}$ denote the Euler-Lagrange derivatives of u with respect to ψ_α and respectively to $h_{\alpha\beta}$. If $u^{(l)}$ is a homogeneous polynomial of order $l > 0$ in the fields and their spacetime derivatives, then $Nu^{(l)} = lu^{(l)}$. Using (104), (107)–(108) and (110) we infer that

$$N\lambda_1 = -\tfrac{1}{2}\left(\bar{B}^{\mu\nu}\partial_{[\mu}\psi_{\nu]} + \bar{A}^{\rho\alpha\beta}\partial_{[\rho}h_{\alpha]\beta}\right) + \partial_\mu \bar{s}^\mu. \tag{111}$$

Now, we expand λ_1 like

$$\lambda_1 = \sum_{l>0}\lambda_1^{(l)}, \tag{112}$$

where $N\lambda_1^{(l)} = l\lambda_1^{(l)}$, such that

$$N\lambda_1 = \sum_{l>0} l\lambda_1^{(l)}. \tag{113}$$

Comparing the relation (111) with (113), we conclude that $\bar{B}^{\mu\nu}$ and $\bar{A}^{\rho\alpha\beta}$ inherit some decompositions similar to (112)

$$\bar{B}^{\mu\nu} = \sum_{l>0} \bar{B}^{\mu\nu}_{(l-1)}, \qquad \bar{A}^{\rho\alpha\beta} = \sum_{l>0} \bar{A}^{\rho\alpha\beta}_{(l-1)}. \tag{114}$$

Inserting (114) in (111) and comparing the resulting expression with (113) we deduce that

$$\lambda_1^{(l)} = -\tfrac{1}{2l}\left(\bar{B}^{\mu\nu}_{(l-1)}\partial_{[\mu}\psi_{\nu]} + \bar{A}^{\rho\alpha\beta}_{(l-1)}\partial_{[\rho}h_{\alpha]\beta}\right) + \partial_\mu \bar{s}^\mu_{(l)}. \tag{115}$$

Replacing (115) in (112), we find that

$$\lambda_1 = -\tfrac{1}{2}\left(B^{\mu\nu}\partial_{[\mu}\psi_{\nu]} + A^{\rho\alpha\beta}\partial_{[\rho}h_{\alpha]\beta}\right) + \partial_\mu z^\mu, \tag{116}$$

where

$$B^{\mu\nu} = \sum_{l>0} \tfrac{1}{l}\bar{B}^{\mu\nu}_{(l-1)}, \qquad A^{\rho\alpha\beta} = \sum_{l>0} \tfrac{1}{l}\bar{A}^{\rho\alpha\beta}_{(l-1)}. \tag{117}$$

Using (116) we obtain that

$$\begin{aligned}
\gamma\lambda_1 \;=\; & -\tfrac{1}{2}\Bigg\{\bar{C}\partial_\lambda\left(\frac{\partial^L B^{\mu\nu}}{\partial\bar{\psi}_\lambda}\partial_{[\mu}\psi_{\nu]} - \frac{\partial^L A^{\mu\nu\rho}}{\partial\bar{\psi}_\lambda}\partial_{[\mu}h_{\nu]\rho}\right) \\
& +2\left(\frac{\partial B^{\mu\nu}}{\partial h_{\alpha\beta}}\partial_{[\mu}\psi_{\nu]} + \frac{\partial A^{\mu\nu\rho}}{\partial h_{\alpha\beta}}\partial_{[\mu}h_{\nu]\rho}\right)\sigma_{\alpha\beta}\xi \\
& -2\left[\partial_\alpha\left(\frac{\partial B^{\mu\nu}}{\partial h_{\alpha\beta}}\partial_{[\mu}\psi_{\nu]} + \frac{\partial A^{\mu\nu\rho}}{\partial h_{\alpha\beta}}\partial_{[\mu}h_{\nu]\rho}\right)\right]\eta_\beta \\
& -\left(\partial_\beta A^{\rho\alpha\beta}\right)\partial_{[\rho}\eta_{\alpha]}\Bigg\} + \partial_\mu t^\mu.
\end{aligned} \tag{118}$$

Comparing (118) with (100) and taking into account the fact that $\bar{C}$, ξ, η_β and $\partial_{[\rho}\eta_{\alpha]}$ are independent elements of pure ghost number equal to one of the basis in the space of polynomials in the ghosts, we find that the tensors $B^{\mu\nu}$ and $A^{\rho\alpha\beta}$ are restricted to fulfill the conditions:

$$\partial_\lambda\left(\frac{\partial^L B^{\mu\nu}}{\partial\bar{\psi}_\lambda}\partial_{[\mu}\psi_{\nu]} - \frac{\partial^L A^{\mu\nu\rho}}{\partial\bar{\psi}_\lambda}\partial_{[\mu}h_{\nu]\rho}\right) = 0, \tag{119}$$

$$\left(\frac{\partial B^{\mu\nu}}{\partial h_{\alpha\beta}}\partial_{[\mu}\psi_{\nu]} + \frac{\partial A^{\mu\nu\rho}}{\partial h_{\alpha\beta}}\partial_{[\mu}h_{\nu]\rho}\right)\sigma_{\alpha\beta} = 0, \tag{120}$$

$$\partial_\alpha\left(\frac{\partial B^{\mu\nu}}{\partial h_{\alpha\beta}}\partial_{[\mu}\psi_{\nu]} + \frac{\partial A^{\mu\nu\rho}}{\partial h_{\alpha\beta}}\partial_{[\mu}h_{\nu]\rho}\right) = 0, \tag{121}$$

$$\partial_\beta A^{\rho\alpha\beta} = 0. \tag{122}$$

Since $A^{\rho\alpha\beta}$ are nonderivative functions, from the last condition we deduce that they are constant. By covariance arguments they must vanish

$$A^{\rho\alpha\beta} = 0. \tag{123}$$

Inserting (123) in (119)–(121) we arrive at

$$\begin{aligned}
\partial_\lambda \left(\frac{\partial^L B^{\mu\nu}}{\partial \bar{\psi}_\lambda} \partial_{[\mu} \psi_{\nu]} \right) &= 0, && (124) \\
\sigma_{\alpha\beta} \frac{\partial B^{\mu\nu}}{\partial h_{\alpha\beta}} \partial_{[\mu} \psi_{\nu]} &= 0, && (125) \\
\partial_\alpha \left(\frac{\partial B^{\mu\nu}}{\partial h_{\alpha\beta}} \partial_{[\mu} \psi_{\nu]} \right) &= 0. && (126)
\end{aligned}$$

The equation (124) implies that

$$\frac{\partial^L B^{\mu\nu}}{\partial \bar{\psi}_\lambda} \partial_{[\mu} \psi_{\nu]} = \partial_\mu S^{\mu\lambda}, \qquad S^{\mu\lambda} = -S^{\lambda\mu}, \tag{127}$$

for some spinor-tensor $S^{\mu\lambda}$. By direct computation we find

$$\begin{aligned}
\frac{\partial^L B^{\mu\nu}}{\partial \bar{\psi}_\lambda} \partial_{[\mu} \psi_{\nu]} &= \partial_\mu \left(\frac{\partial^L B^{\mu\nu}}{\partial \bar{\psi}_\lambda} \psi_\nu - \frac{\partial^L B^{\lambda\nu}}{\partial \bar{\psi}_\mu} \psi_\nu \right) \\
&\quad + \partial_\mu \left(\frac{\partial^L B^{\mu\nu}}{\partial \bar{\psi}_\lambda} \psi_\nu + \frac{\partial^L B^{\lambda\nu}}{\partial \bar{\psi}_\mu} \psi_\nu \right) \\
&\quad -2 \left(\partial_\mu \frac{\partial^L B^{\mu\nu}}{\partial \bar{\psi}_\lambda} \right) \psi_\nu. \qquad (128)
\end{aligned}$$

Comparing (127) with (128) we infer that the last two terms in the right-hand side of (128) must vanish

$$\begin{aligned}
\partial_\mu \left(\frac{\partial^L B^{\mu\nu}}{\partial \bar{\psi}_\lambda} \right) &= 0, && (129) \\
\partial_\mu \left(\frac{\partial^L B^{\mu\nu}}{\partial \bar{\psi}_\lambda} \psi_\nu + \frac{\partial^L B^{\lambda\nu}}{\partial \bar{\psi}_\mu} \psi_\nu \right) &= 0. && (130)
\end{aligned}$$

As $B^{\mu\nu}$ contains no derivatives, the equation (129) gives

$$\frac{\partial^L B^{\mu\nu}}{\partial \bar{\psi}_\lambda} = C^{\lambda\mu\nu}, \qquad C^{\lambda\mu\nu} = -C^{\lambda\nu\mu}, \tag{131}$$

for some constant matrices $C^{\lambda\mu\nu}$. Substituting (131) into (130), we obtain the equation $\partial_\mu \left(\left(C^{\lambda\mu\nu} + C^{\mu\lambda\nu}\right)\psi_\nu\right) = 0$, which further implies

$$C^{\lambda\mu\nu} = -C^{\mu\lambda\nu}, \tag{132}$$

so the objects $C^{\lambda\mu\nu}$ are completely antisymmetric in their Lorentz indices. In consequence, from (131) we get

$$B^{\mu\nu} = \bar{\psi}_\lambda C^{\lambda\mu\nu}. \tag{133}$$

From (133) we find that $\partial B^{\mu\nu}/\partial h_{\alpha\beta} = 0$, so the solution (133) verifies also the equations (125) and (126). In the meantime, the general form of the constant matrices $C^{\lambda\mu\nu}$ reads as $C^{\lambda\mu\nu} = k\gamma^{[\lambda}\gamma^\mu\gamma^{\nu]}$, with k an arbitrary numerical constant, such that

$$B^{\mu\nu} = k\bar{\psi}_\lambda\gamma^{[\lambda}\gamma^\mu\gamma^{\nu]}. \tag{134}$$

Introducing the solutions (123) and (134) in (116), it follows that

$$\lambda_1 \propto \bar{\psi}_\lambda\gamma^{[\lambda}\gamma^\mu\gamma^{\nu]}\partial_{[\mu}\psi_{\nu]}. \tag{135}$$

We are now left with investigating the solutions to the equations (101). Taking into account the hypothesis on the preservation of the number of derivatives on each field, we obtain that

$$\begin{aligned}
\lambda_2 &= \lambda_2\left(\psi, h, \partial h, \partial h\right), && (136)\\
\lambda_3 &= \lambda_3\left(\psi, h, \partial h, \partial h, \partial h\right), && (137)\\
\lambda_4 &= \lambda_4\left(\psi, h, \partial h, \partial h, \partial h, \partial h\right), && (138)
\end{aligned}$$

which signifies that for the values $k = 2, 3, 4$ the functions λ_k depends only on the undifferentiated Rarita-Schwinger field, on the undifferentiated Weyl field, and also on k spacetime derivatives of order one of the Weyl field. If we make the notations

$$\tilde{A}_k^{\alpha\beta} = \frac{\delta\lambda_k}{\delta h_{\alpha\beta}}, \qquad \tilde{B}_k^\mu = \frac{\partial^R\lambda_k}{\partial\psi_\mu}, \qquad k = 2, 3, 4, \tag{139}$$

on behalf of the formulas (136)–(138) we deduce that

$$\begin{aligned}
\tilde{B}_2^\mu &= \tilde{B}_2^\mu\left(\psi, h, \partial h, \partial h\right), && (140)\\
\tilde{B}_3^\mu &= \tilde{B}_3^\mu\left(\psi, h, \partial h, \partial h, \partial h\right), && (141)\\
\tilde{B}_4^\mu &= \tilde{B}_4^\mu\left(\psi, h, \partial h, \partial h, \partial h, \partial h\right). && (142)
\end{aligned}$$

By means of the definitions (20), we infer that the equations (101) are satisfied if the Euler-Lagrange derivatives $\tilde{A}_k^{\alpha\beta}$ and $\tilde{B}_k^{\mu}$ are subject to the equations

$$\partial_\mu \tilde{B}_k^{\mu} = 0, \tag{143}$$

$$\partial_\alpha \tilde{A}_k^{\alpha\beta} = 0, \qquad \tilde{A}_k^{\alpha\beta}\sigma_{\alpha\beta} = 0. \tag{144}$$

The solutions to the equations (143) are given by

$$\tilde{B}_k^{\mu} = \partial_\nu \tilde{B}_k^{\nu\mu}, \qquad \tilde{B}_k^{\nu\mu} = -\tilde{B}_k^{\mu\nu}, \tag{145}$$

while the solutions to (144) are not important in what follows and will not be considered here. On the one hand, the relations (140)–(142) show that $\tilde{B}_k^{\mu}$ do not depend on the derivatives of the Rarita-Schwinger field, and, on the other hand, the equations (145) imply that $\tilde{B}_k^{\mu}$ do depend on these derivatives. As a consequence, the solutions to the equations (143) are merely constant, i.e.,

$$\tilde{B}_k^{\mu} = C_k^{\mu}. \tag{146}$$

Due to the fact that $\tilde{B}_k^{\mu}$ are spinor-vectors and since the present field spectrum does not allow the construction of constant spinor-vectors, it results that the sole solution to the equations (143) is vanishing

$$\tilde{B}_k^{\mu} = 0. \tag{147}$$

Replacing (147) in the latter relation in (139), we reach the conclusion that the quantities $(\lambda_k)_{k=\overline{2,4}}$ do not depend on the Rarita-Schwinger field and, consequently, they cannot describe cross-couplings between the Weyl graviton and the Rarita-Schwinger field, as required, so we can take

$$\lambda_k = 0, \qquad k = 2, 3, 4.$$

In this manner we arrive at the final solution

$$a_0^{(\mathrm{int})} = a_0'^{(\mathrm{int})} + a_0''^{(\mathrm{int})} = \lambda_1 \propto \bar{\psi}_\lambda \gamma^{[\lambda}\gamma^{\mu}\gamma^{\nu]}\partial_{[\mu}\psi_{\nu]} \tag{148}$$

to the equation (94).The above $a_0^{(\mathrm{int})}$ does not describe cross-couplings between the Weyl graviton and the Rarita-Schwinger field. Moreover, it neither produces self-interactions of the Rarita-Schwinger field since it is proportional with the free Lagrangian of this theory, and, accordingly, it must be discarded from the first-order deformation. Thus, there is no nontrivial possibility to

couple the Weyl graviton to the massless Rarita-Schwinger field by means of a first-order deformation that reduces to its antighost number zero component under the working hypotheses invoked in this paper. Thus, the conclusion of this section is that the first-order deformation vanishes

$$S_1 = 0, \tag{149}$$

so the solutions to the higher-order deformation equations, (28)–(29), etc., also vanish

$$S_2 = S_3 = \cdots = 0. \tag{150}$$

5 Conclusion

In this paper we have investigated the cross-couplings that can be introduced between the Weyl graviton and the massless Rarita-Schwinger field from the BRST formalism point of view. Thus, under the general conditions of locality, smoothness, (background) Lorentz invariance, Poincaré invariance and preservation of the number of derivatives with respect to each field (the last hypothesis was made only in antighost number zero), we have proved that there are no such cross-couplings. The only deformations that can be introduced in relation with the free model under study are represented by the self-interactions of Weyl gravity, since there are no consistent, nontrivial self-interactions of the massless Rarita-Schwinger field (see, for instance, the consistency arguments invoked in [26]).

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